Estimation of Non-market Values in the Presence of Social Network Effects

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Abstract

Stated preference (SP) valuation methods are often the only available approach to estimation of non-market values. Among elicitation formats, dichotomous choice is commonly used and has potentially desirable incentive properties. This paper shows that yes/no responses are not independent when non-market values are influenced by social network effects. However, the empirical literature has yet to attempt estimation of non-market values explicitly accommodating network effects. We investigate the statistical properties of estimates of mean willingness to pay obtained through standard econometric approaches that ignore social networks. Monte Carlo experiments, with different types of simulated and real world social networks, indicate that failure to account for network effects leads to underestimation of non-market values.

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1 Introduction

The ability to estimate non-market values has made economists a fundamental asset in formulating and analyzing public policies. It is common for policymakers to face choices that comprise trade-offs involving non-market goods. In such circumstances, the non-market valuation literature provides guidance to estimation of non-market values, which is essential to benefit-cost analyses.

Stated preference methods are often the only available approach to evaluate public projects. Using SP policymakers can study the provision of non-market goods in terms that are different from what is observed in revealed behavior data. Surveys allow researchers to create scenarios and obtain monetary values for different non-market goods, with different provision rules. This enables policymakers to design better projects, aiming at maximum welfare for a given budget constraint. As highlighted by Carson and Hanemann (2005, pg. 825), "much of the usefulness of conducting a SP study has nothing to do with explicitly obtaining an estimate of monetary value". They argue that SP data provide valuable information about the distribution of values of a public project, and how this distribution varies with variables such as demographics and characteristics of the project.

Under the simplest and most commonly used SP question format, the respondent is offered a binary choice between two alternatives. First, she can vote for maintaining the status quo policy. Second, she can vote in favor of an alternative policy at a specified cost. In addition to its simplicity, Carson and Groves (2007) demonstrate that under certain assumptions dichotomous-choice questions framed as referendum vote have desirable properties of incentive compatibility.[1]

Two frameworks provide the econometric foundation for dichotomous choice SP. Hanemann (1984) and Hanemann and Kanninen (1996) develop random utility models (RUM) based on differences in the indirect utility functions, comparing utility before and after the proposed policy.[2] Cameron and James (1987) and Cameron (1988) construct random will-

[1] The literature early on recognizes advantages of the binary choice format. The Gibbard-Satterwaite theorem states that multinomial choice questions (i.e. the respondent is offered $k > 2$ alternatives) can not be incentive compatible without placing restrictions on the respondent's utility (see Gibbard (1973) and Satterthwaite (1975)).

[2] This approach is often referred to as the Hanemann's approach.

ingness to pay (WTP) models based on differences in the expenditure function.[3] McConnell (1990) demonstrates that, under a reasonable set of assumptions, these frameworks are dual to one another.

The utility-theoretic interpretation of the yes/no responses in both RUM and WTP frameworks are constructed based on agents in social isolation. However, Neilson and Wichmann (WP 2013) develop a utility-based model in which the value of non-market goods can be affected by characteristics of the respondents' social network. If a respondent is altruistic and cares about non-market goods that benefit her friends, the utility of friends will influence the respondent's voting decision in a SP survey. Similarly, friends may also be altruistic and the utility of friends of friends may influence the voting behavior of the respondent's friends. This leads to network effects.[4]

The social structure of the population of interest can be an important determinant of the shape of the WTP distribution. With social network effects, a respondent's vote is influenced by her friends' votes. Her friends' votes, in turn, are influenced by the respondent's vote. This generates a reflection problem (see Manski (1993, 2000)). Therefore, an approach to recover non-market values from yes/no SP responses must be based on the estimation of a discrete choice model with network dependence. This issue has been, however, overlooked by the empirical non-market valuation literature.

This paper investigates the consequences of ignoring social network effects for dichotomous choice SP. We built on the work of Cameron and James (1987) to develop a random WTP model with social networks.[5] The model is directly related to the theory developed by Neilson and Wichmann (WP 2013) . It can be easily shown that a standard WTP model utilized in Cameron-like approaches is a especial case of the more general network formulation developed in this paper. Our econometric specification is similar to a spatial autoregressive dependent variable model (SAL). The model is constructed by replacing the traditional

[3]This approach is often referred to as the Cameron's approach.

[4]As discussed by Neilson and Wichmann (WP 2013), altruism is only one possible channel for the interdependence of respondents' utilities. Another reason might be joint consumption. If the utility of consuming a good in social isolation is different from the utility of consuming the same good with friends, joint consumption may also lead to network effects.

[5]Our econometric model builds on Cameron's expenditure difference formulation of dichotomous choice responses, however, the results can be extended to Hanemann's utility difference formulation.

weighting matrix of a spatial econometric model, usually assumed to be a distance matrix, by a row stochastic matrix that represents the social network.

The main result of the paper is that, in the presence of social network effects, standard dichotomous choice estimation approaches are inconsistent. Specifically, when utilities are influenced by social networks, the standard RUM and WTP models are misspecified. Hence, maximum likelihood estimation of the parameters of these models is inconsistent because estimation is based on a misspecified likelihood function. The inconsistency arises from the heteroskedasticity induced by the network dependence in the discrete choice model.

If social networks are an important determinant of WTP, then estimation of non-market values is more challenging than is currently believed to be. A proper valuation approach must account for the heteroskedasticity induced by the network dependence. Moreover, for efficiency to be achieved, estimation must also use all the information contained in the non-diagonal variance-covariance structure of network models.[6]

The paper reports results of a Monte Carlo experiment designed to explore how the bias of standard approaches is influenced by characteristics of the social network and the intensity of the network effect. In an initial control simulation without network effects we find that the traditional mean WTP estimation is indeed consistent. Next, we find that when WTP is influenced by Erdos-Renyi networks, the performance of the standard estimation approach is negatively affected by the strength of the network effect, but is not influenced by the density of the network. We also find that, in networks with high correlation between respondents' importance (i.e. a measure of "popularity" of respondents) and private WTP, the estimation bias is very sensitive to strong network effects. Specifically, the coefficient of variation of of the distribution of traditional estimates of mean WTP is 308% in such an environment. Finally, the experiment shows that traditional mean WTP estimates are not reliable when the data is generated using real world social networks. We use data collected by Banerjee et al. (2011) of social networks in three rural villages of India to perform three Monte Carlo experiments. We find that, although the standard approach is theoretically inconsistent, it performs relatively well for simulations with villages 1 and 2. However, the distribution of

[6] Refer to Fleming (2004) for a detailed discussion about spatial models with binary dependent variable. Refer to Bramoullé et al. (2009) for estimation of network models with continuous dependent variable.

estimated mean WTP shifts to the left when the network of village 3 generates the data. Network-level statistics are not able to explain this phenomenon. Further research is needed to formally identify the effects of the characteristics of real world social networks on standard estimates of mean WTP.

Our results indicate that social networks place an additional layer of complexity to benefit transfer. With social network effects, the welfare generated by non-market goods provision takes place in the context of a particular social structure. A similar public project in a different location will probably be implemented in a very different social network. This type of difficulty is usually associated with revealed preference studies (because observed behavior is a function of the market structure) and should also be recognized in SP studies.

The remainder of the paper is organized as follows. Section 2 presents a network model of random willingness to pay. Section 3 discusses the consequences of ignoring social networks in dichotomous choice SP. Section 4 provides a Monte-Carlo investigation. Section 5 concludes.

2 A Stochastic Model of Willingness to Pay in Social Networks

There are n agents arranged in a social network. Let $\mathbf{A}$ be an $n \times n$ row stochastic matrix that represents the network. Diagonal elements of $\mathbf{A}$ are equal to zero while off-diagonal elements are either $a_{ij} \neq 0$, if agent i is influenced by agent j, or $a_{ij} = 0$ otherwise. An element a_{ij} (for $i \neq j$) is the weight of i's social connection to agent j. Notice that symmetry of $\mathbf{A}$ (typically assumed in spatial models) is not required, hence, $\mathbf{A}$ denotes a directed network.[7]

Agents have utility over a non-market good that is provided to the entire network. The goal is to evaluate a public project that yields a discrete increase in the provision of this non-market good. Assume that the willingness to pay for the public project is given by

$$\mathbf{WTP}^* = \alpha \mathbf{i} + \mathbf{X}\gamma + \beta \mathbf{A}\,\mathbf{WTP}^* + \epsilon, \tag{1}$$

[7] In spatial models, the matrix $\mathbf{A}$ define the spatial lags of the left-hand side variable and it is typically assumed to be row stochastic and symmetric, e.g. a matrix of relative distances (see LeSage (1999)).

where $\mathbf{WTP}^*$ is an $n \times 1$ vector of unobserved willingness to pay, $\mathbf{i}$ is an $n \times 1$ vector of ones, $\mathbf{X}$ is an $n \times k$ matrix of k exogenous variables, and ϵ is an $n \times 1$ vector of i.i.d. normal errors, i.e. $\epsilon \overset{iid}{\sim} N(0, \sigma_\epsilon^2)$.[8] The i-th row of $\mathbf{A}\,\mathbf{WTP}^*$ is a weighted average of i's connected agents' willingness to pay. The intercept of the model is α. γ is a $k \times 1$ parameter vector that captures the effects of the agents' own characteristics $\mathbf{X}$ on $\mathbf{WTP}$. β is the network effect parameter that captures the effect of connected agents' $\mathbf{WTP}$ on own $\mathbf{WTP}$.

The random WTP model is closely related to the network model developed by Neilson and Wichmann (WP 2013). To see this, recall that the respondents' social utility profile under the status quo provision level of the public good (g^0) is

$$\mathbf{v}(g^0) = (\mathbf{I} - \mathbf{\Lambda})\mathbf{u}(g^0) + \mathbf{\Lambda}\mathbf{A}\mathbf{v}(g^0), \tag{2}$$

and the social utility profile under the policy provision level of the public good (g^1) is

$$\mathbf{v}(g^1) = (\mathbf{I} - \mathbf{\Lambda})\mathbf{u}(g^1) + \mathbf{\Lambda}\mathbf{A}\mathbf{v}(g^1). \tag{3}$$

Subtracting (2) from (3) we obtain

$$\mathbf{C}^{netwotk} = (\mathbf{I} - \mathbf{\Lambda})\mathbf{C}^{private} + \mathbf{\Lambda}\mathbf{A}\mathbf{C}^{network}.$$

The deterministic term $\alpha\mathbf{i} + \mathbf{X}\gamma$ in equation (1) corresponds to private WTP. The innovation of model (1) is the introduction of the network term $\beta\mathbf{A}\,\mathbf{WTP}^*$ that predicts the impact of connected agents on own valuation. A hypothesis test of the null $H_0 : \beta = 0$ is an empirical test of the existence of social network effects.

Policymakers are interested in estimates of $\boldsymbol{\theta} = (\alpha, \gamma, \beta)$ to better design policy. Features of the public project are often included in $\mathbf{X}$ along with respondents' characteristics. With dichotomous choice SP, the econometrician does not observe WTP. Thus, estimates of $\boldsymbol{\theta}$ are obtained through observed voting behavior on "take-it-or-leave-it" survey questions. Assuming that the structural model (1) determines WTP, section 3 discusses the properties

[8] We assume strict exogeneity of $\mathbf{X}$, i.e. $\mathrm{E}(\epsilon|\mathbf{X}) = 0$.

of estimates of $\boldsymbol{\theta}$ when estimation is guided by the standard WTP approach that does not account for social network effects.

3 Consequences of Ignoring Network Effects

The structural model (1) describes a respondent's latent WTP. Yes/no survey responses are, however, determined by the reduced form of model (1). The reduced form equation is

$$\mathbf{WTP}^* = (\mathbf{I} - \beta\mathbf{A})^{\mathbf{-1}}\alpha\mathbf{i} + (\mathbf{I} - \beta\mathbf{A})^{\mathbf{-1}}\mathbf{X}\gamma + \eta \tag{4}$$

where $\eta = (\mathbf{I} - \beta\mathbf{A})^{-1}\epsilon$. We make the standard assumption that $\mid \beta \mid < 1$, thus, the matrix ($\mathbf{I}$ - $\beta\mathbf{A}$) is invertible.[9]

The paper focuses on the expenditure difference formulation of dichotomous choice responses (Cameron's approach), however our results can be extended to the utility difference formulation (Hanemann's approach). In a dichotomous choice study we present respondent i with a "take-it-or-leave-it" offer to vote yes or no for the public project at cost t_i. Assuming that respondents truthfully answer the survey we observe the following data.

$$y_i = \begin{cases} 1 & \text{if} \quad WTP_i^* \geq t_i \\ 0 & \text{if} \quad WTP_i^* < t_i \end{cases}$$

The marginal probabilities are obtained as follows.

$$\begin{aligned} Prob\Big(y_i = 1|\mathbf{X}\Big) &= Prob\Big(\big[(\mathbf{I} - \beta\mathbf{A})^{-1}\alpha\mathbf{i}\big]_i + \big[(\mathbf{I} - \beta\mathbf{A})^{-1}\mathbf{X}\gamma\big]_i + \eta_i > t_i\Big) \\ &= Prob\Big(\eta_i > t_i - \big[(\mathbf{I} - \beta\mathbf{A})^{-1}\alpha\mathbf{i}\big]_i - \big[(\mathbf{I} - \beta\mathbf{A})^{-1}\mathbf{X}\gamma\big]_i\Big) \end{aligned} \tag{5}$$

The network effect introduces an interdependence in WTP_i^* and, as a result, the reduced form error η is distributed by a n-dimensional multivariate normal, with mean zero and

[9] If $\mid \beta \mid < 1$, then $(\mathbf{I} - \beta\mathbf{A})$ is a strictly diagonally dominant matrix and, by the Levy-Desplanques theorem, cannot be singular (see Taussky (1949), Theorem I).

variance-covariance matrix equal to

$$E(\eta\eta') = (\mathbf{I} - \beta\mathbf{A})^{-1}(\mathbf{I} - \beta\mathbf{A})^{-1\prime}\sigma_\epsilon^2. \tag{6}$$

Denote the i-th diagonal element of (6) as $\sigma^2_{\eta i}(\beta)$, and construct the standardized variable $z_i = \eta_i/\sigma^2_{\eta i}(\beta)$. We can re-write (5) as

$$Prob\Big(y_i = 1|\mathbf{X}\Big) \quad = \quad Prob\Big(z_i > \frac{t_i - \big[(\mathbf{I} - \beta\mathbf{A})^{-1}\alpha\mathbf{i}\big]_i - \big[(\mathbf{I} - \beta\mathbf{A})^{-1}\mathbf{X}\gamma\big]_i}{\sigma^2_{\eta i}(\beta)}\Big). \tag{7}$$

Equation (7) highlights the econometric challenge of the estimation of $\boldsymbol{\theta}$. With no network effects ($\beta = 0$) equation (7) simplifies to

$$Prob\Big(y_i = 1|\mathbf{X}\Big) \quad = \quad Prob\Big(z'_i > \frac{t_i - \big[\alpha\mathbf{i} - \mathbf{X}\gamma\big]_i}{\sigma^2_\epsilon}\Big), \tag{8}$$

where $z'_i = \epsilon_i/\sigma^2_\epsilon$ is the standard normal random variable. Equation (8) is the basis for estimation of WTP through standard maximum likelihood approaches. However, the procedure is based on the diagonal variance-covariance matrix $\sigma^2_\epsilon\mathbf{I}$. With independent errors, the likelihood of observing the data is

$$\Pi_{i=1}^n \int_{-\infty}^{a_i} \phi(z_i)dz_i, \tag{9}$$

where ϕ is the standard normal pdf, and $a_i = [1 - 2y_i][t_i - (\alpha\mathbf{i} - \mathbf{X}\gamma)_i]/\sigma_\epsilon$.

The variance-covariance matrix of the model is described by (6) when there are network effects. The off-diagonal elements of $\mathrm{E}(\eta\eta')$ are not zero, errors are correlated and distributed according to a n-dimensional normal. The likelihood of observing the data is now

$$\int_{-\infty}^{a_1} \ldots \int_{-\infty}^{a_n} f(\mathbf{z})d\mathbf{z}, \tag{10}$$

where f is the multivariate normal governing $\mathbf{z}$.[10]

In summary, the reduced form model that explains yes/no responses has a heteroskedastic

[10] Refer to Fleming (2004) for additional details.

error term. The heteroskedasticity is induced by the social network structure. This leads to the paper's proposition.

Proposition 1. *If respondents consider their social networks when valuing public projects (i.e. $\beta \neq 0$), then the standard approach for estimation of non-market values is inconsistent.*

Proof. Standard estimation uses optimization techniques to maximize the logarithm of the likelihood function (9). The log-likelihood function is

$$\mathcal{L} = \sum_{i=1}^{n} \left\{ y_i \log\left[1 - \Phi\left((t_i - \left[\alpha \mathbf{i} - \mathbf{X}\gamma\right]_i)/\sigma_\epsilon\right)\right] + (1 - y_i)\log\left[\Phi\left((t_i - \left[\alpha \mathbf{i} - \mathbf{X}\gamma\right]_i)/\sigma_\epsilon\right)\right]\right\},$$

where Φ is the standard normal cdf. However, with network effects, i.e. $\beta \neq 0$, the function $\mathcal{L}$ is misspecified and estimation must be based on (10), and not on (9). □

In words, the network structure introduces heteroskedasticity to the WTP model. Hence, standard approaches that ignore network effects are inappropriate for estimation of $\boldsymbol{\theta}$ because they are not robust to unspecified heteroskedasticity. Clearly, estimates of mean WTP are also inconsistent. Two facts make this an unsettling result. First, to the best of our knowledge, the valuation literature has failed to attempt estimation of non-market values accounting for social network effects. Second, the norm of the profession is to perform SP using dichotomous choice data given the incentive compatibility properties of this elicitation mechanism.

The paper's proposition implies that estimation of non-market values based on dichotomous choice data must use techniques for estimating spatially dependent discrete choice models.[11] In general, these estimators can be divided in two major groups: heteroskedastic estimators, and full information estimators. Heteroskedastic estimators address the spatial (or network) dependence issue and provide consistent estimates of the parameters of the likelihood function. However, consistency relies on the assumption that the off-diagonal elements of the variance-covariance matrix are zero.

[11] Refer to LeSage (1999), Fleming (2004), and Franzese Jr and Hays (2008) for a discussion of the estimation challenges related to discrete choice models with spatial dependence.

Full spatial information estimators account for the off-diagonal variance-covariance terms. These terms are usually not zero in real-world applications with complex social networks. This highlights a major practical difference between maximum likelihood estimation with and without networks. The variance structure of the network model does not allow the simplification of the multivariate normal into the product of univariate normal distributions. Valuation of the likelihood function is complex because involves integrating the joint distribution over n dimensions.

According to Proposition 1, standard estimation approaches deliver inconsistent estimates of mean WTP when non-market values are influenced by social networks. Since the dichotomous choice elicitation format has been so widely used, it is important to understand how the structure of a network influences traditional estimates. Next section investigates this question.

4 Monte Carlo Experiments

The Monte Carlo experiments examine the random WTP model (1). Our goal is to explore the performance of standard estimation approaches when the social network term $\beta \mathbf{A}\mathbf{Y}^*$ is ignored. To do this, we estimate mean WTP using a maximum likelihood probit regression model as discussed by Cameron and James (1987). The dependent variable is a binary indicator for the yes/no response. The policy cost is included on the right hand side among the explanatory variables $\mathbf{X}$. The parameters of the latent WTP model are recovered from the probit estimates as demonstrated by Cameron and James (1987).

We expect that the standard estimation approach performs poorly when $\beta \neq 0$ and delivers inconsistent estimates of mean WTP (see proposition 1). Our Monte Carlo experiments aim to explore how the WTP bias reacts to changes in the strength of the network effect, β, and to changes of the type of network structure.

The setup of each Monte Carlo experiment is the following. We consider a population of size $n = 300$. We use the reduced form equation (4) to construct a vector of "true" WTP (i.e. a vector in which element i is the WTP of respondent i). Next, we generate 1000 Monte Carlo samples by re-sampling the error term (the Monte Carlo samples are replications of the "true" model). We use the standard approach to estimate mean WTP in each Monte Carlo sample. Specifically, for every replication $j = 1, ..., 1000$, each respondent $i = 1, ..., 300$ votes yes or no for the project at cost t_i. A "yes" response ($y_i = 1$) is observed if $WTP_i \geq t_i$, "no" ($y_i = 0$) is observed otherwise. The cost t_i faced by respondent i is randomly selected from the set of the deciles of the original "true" WTP distribution.

The variables of the right hand side of equation (4) are determined as follows. For simplicity, $\mathbf{X}$ is assumed to be a single variable. The explanatory variable is constructed to be orthogonal to the error term ϵ and the network matrix $\mathbf{A}$ as follows. Define $\tilde{\mathbf{X}}$ as a vector with elements increasing in equal increments from $\tilde{X}_1 = 0$ to $\tilde{X}_n = 1$. The vector $\mathbf{X}$ is a scrambled version of the vector $\tilde{\mathbf{X}}$. The same vector $\mathbf{X}$ is used in every Monte Carlo sample.

The parameters values are fixed as follows.

$$\alpha = 2 \qquad\qquad \gamma = 4$$

This implies that the deterministic part of the unobserved WTP (i.e. $\alpha\mathbf{i} + \mathbf{X}\gamma$) ranges from \$2 to \$6. We explore four values for β. First we assume $\beta = 0$ representing no network effects. We expect the standard approach to perform very well in this model. Next we set β equal to 0.25, 0.50, and 0.75, representing environments of increasing social network effects.

Three types of networks are explored. First we study Erdos-Renyi networks in which links are i.i.d. and each pair of respondents is connected with fixed probability d. Second we explore networks with a strong correlation between private WTP and respondents' "popularity" (i.e. agent importance of Neilson and Wichmann (WP 2013)). Finally, we investigate estimation of WTP using real-world social network data collected by Banerjee et al. (2011).

As a result, we perform 37 Monte Carlo experiments. The number of experiments is determined by the number of networks (eight in section 4.1, one in section 4.2, and three in section 4.3) and the number of βs (0, 0.25, 0.50, and 0.75). Hence, 37 vectors of "true" WTP are generated according to the reduced form equation (4).[12] The same vector of errors ϵ, with elements drawn independently from a standard normal distribution, is used to construct the "true" WTP vector of all experiments. Therefore, as previously described, we generate 1000

[12] Number of experiments is equal to the number of networks (12) times the number of βs that are different from zero (3), plus the model with no network effect ($\beta = 0$).

Monte Carlo samples for each one of the 37 experiments. The size of each sample is equal to $n = 300$. We construct these samples by re-drawing 1000 vectors ϵ. We use the same 1000 error vectors, together with the fixed vector $\mathbf{X}$, in all experiments. Hence, the differences between the experiments come exclusively from variations of β and $\mathbf{A}$.

To evaluate the performance of the standard approach we first compute the mean WTP Root-Mean-Square-Error (RMSE). For each Monte Carlo experiment, the RMSE is computed as

$$RMSE = \sqrt{\frac{\sum_{j=1}^{r}\left(\widehat{E[WTP]}_j - E[WTP]\right)^2}{r}},$$

where r is the number of Monte Carlo samples (or replications), $\widehat{E[WTP]}_j$ is the standard prediction about mean WTP in replication j, and E[WTP] is the "true" mean WTP. Specifically, the WTP prediction is given by

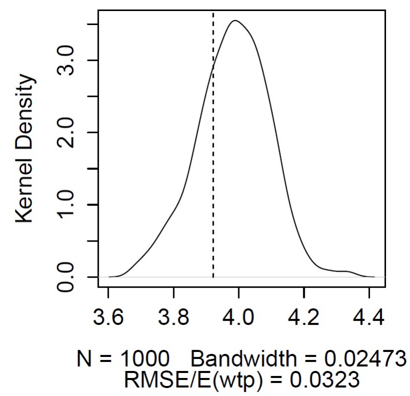
$$\widehat{E[WTP]}_j = \frac{\sum_{i=1}^{n}\widehat{WTP}_{ij}}{n},$$

where $\widehat{WTP}_{ij}$ is the estimate of WTP of respondent i in replication j.[13] The RMSE is measured in dollar units and can, therefore, be directly compared with the "true" mean WTP. The normalization RMSE/E[WTP] is particularly informative and measures the average bias of the standard approach in percentage terms. We refer to this value as the *coefficient of variation* of the distribution of estimated WTP.

Let us first evaluate the performance of the traditional estimator when $\beta = 0$. In this case, there is no network effect and the likelihood function is correctly specified. Figure 1 presents the kernel density function of the $\widehat{E[WTP]}$ obtained through the $r = 1000$ trials. The vertical line denotes the "true" value of mean WTP. As expected, the standard estimator performs well and the coefficient of variation is only 0.0323, i.e. less than 5%. Bellow we examine situations in which $\beta \neq 0$.

[13] WTP_{ij} is obtained by plugging the estimates of $\boldsymbol{\theta}$ in equation (4) and taking the conditional expectation. Notice that even thought η has a complex variance structure, η is a mean zero error.

Figure 1: Kernel Estimates for MC Trials with No Network Effect



4.1 Erdos-Renyi Networks

Erdos-Renyi networks are a natural starting point for the simulations with $\beta \neq 0$. These networks assume that there is a fixed set of nodes (i.e. $n = 300$). Each link is formed with a given probability d, and the formation of links is independent. Let the network *density* be the ratio of actual number of links over the maximum possible number of links (i.e. the relative fraction of existing links). Clearly, the expected density of Erdos-Renyi networks is equal to the probability of connection d.

We perform 24 simulations using Erdos-Renyi networks.[14] We explore four low density networks (d equal to 0.025, 0.050, 0.075, and 0.1), and four high density networks (d equal to 0.2, 0.4, 0.6, and 0.8). Table 1 presents the coefficient of variation of the empirical distribution of $E[\widehat{WTP}]$. Our simulations show that the coefficients of variation hover around 3% when there is a small network effect of $\beta = 0.25$. This result indicates that, when links are independently formed with fixed probability, the standard estimation approach, although theoretically inconsistent, performs remarkably well. In fact, the kernel densities displayed in the first column of Figures 3 and 4 (in the appendix) are similar to the density shown in Figure 1, in which the data generating process has no network effects.

The performance of standard approach is, however, unsatisfactory when β increases to 0.50. The coefficients of variation are approximately 13%. Moreover, the kernel densities

[14] We use 8 networks and 3 βs, totaling 24 simulations.

Table 1: Coefficient of Variation for Erdos-Renyi Simulations (1000 draws).

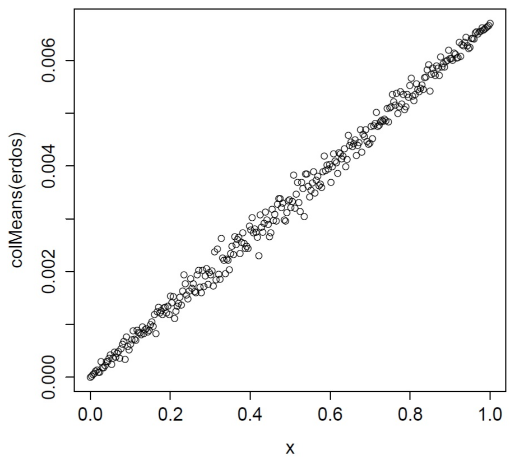
	Network Effect β		
d	0.25	0.50	0.75
0.025	0.0310	0.1303	0.9413
0.050	0.0317	0.1315	0.7649
0.075	0.0311	0.1316	0.6832
0.10	0.0308	0.1285	1.0147
0.20	0.0308	0.1278	0.6597
0.40	0.0303	0.1335	0.8910
0.60	0.0307	0.1299	1.2280
0.80	0.0310	0.1334	0.9040

peak to the left of the true mean WTP suggesting that the standard approach underestimates mean WTP (see second column of Figures 3 and 4). The negative bias is also observed in the Monte Carlo samples with $\beta = 0.75$. In strong network effect conditions, the coefficients of variation are enormous, reaching 123% when $d = 0.6$. The network density seems to have no impact on the mean WTP bias for conditions with $\beta = 0.25$ and $\beta = 0.50$. There is significant fluctuation in the coefficients of variation of the different density simulations when $\beta = 0.75$. However, it is hard to identify a pattern between network density and these errors.

4.2 Matching Importance and Private WTP

In this section, the explanatory variable is constructed to be correlated with the column sums of the network matrix $\mathbf{A}$ as follows. The right hand side variable $\mathbf{X}$ equals $\tilde{\mathbf{X}}$, i.e. a $(n \times 1)$ vector with elements increasing in equal increments from 0 to 1. Links within columns of the network $\mathbf{A}$ are formed with independent probability. The probability of a link in the first column of $\mathbf{A}$ is equal to $X_1 = 0$, the probability of a link in the second column of $\mathbf{A}$ is equal to $X_2 = 0.0033$, the probability of a link in the third column of $\mathbf{A}$ is equal to $X_3 = 0.0066$, and so on. In the last column, the probability of a link is equal to $X_{300} = 1$. The expected density in this network is 0.5. Clearly, there is a strong positive correlation between $\mathbf{X}$ and the importance of respondents. Figure 2 demonstrates this correlation.

Figure 2: Correlation Between Importance and $\mathbf{X}$



Results again indicate that the performance of the standard approach is satisfactory when $\beta = 0.25$ (see Figure 5). The distribution of estimated mean WTP is centered around the true value, and the coefficient of variation is again 3%. As in Erdos-Renyi networks, the performance suffers when β increases. The coefficient of variation increases to 8% when $\beta = 0.50$, and to an impressive 308% when $\beta = 0.75$. This result suggests that a strong correlation between respondents' importance and respondents' exogenous characteristics significantly affects the performance of standard approaches in environments with strong network effects.

4.3 A Real World Social Network

This section uses real network data collected by Banerjee et al. (2011) and made available by the authors online.[15] The data was obtained from a survey of social networks in rural villages of southern Karnataka, a state in India. Individuals were asked detailed questions about the relationships they had with others in the village. This information enables the construction

[15]Data source: http://dvn.iq.harvard.edu/dvn/dv/jpal/faces/study/StudyPage.xhtml?globalId=hdl:1902.1/16559.

of network graphs for each village.[16] This section uses data on the first three villages of the dataset. We denote these villages as Village 1 ($n_1 = 182$ respondents), Village 2 ($n_2 = 195$ respondents), and Village 3 ($n_3 = 292$ respondents). Table 2 presents characteristics of these networks.[17]

Table 2: Real World Networks

	Village 1	Village 2	Village 3
Number of nodes	182	195	292
Average degree	19.08	17.73	17.73
Average path length	2.5734	2.9540	2.8130
Average betweenness	0.0162	0.0169	0.0108
Average closeness	0.0006	0.0002	0.0001
Density	0.0524	0.0355	0.0304
Transitivity	0.1751	0.1777	0.1285
Diameter	5	6	6

Figures 6, 7, and 8 confirm that the standard approach is able to deliver robust estimates in a model with small network effects. When $\beta = 0.25$, the coefficients of variation are below 5% for the Monte Carlo simulations of all three villages. In Village 1, the coefficients of variation are below 10% even when β is high. In Village 2, the coefficient of variation is a little above 10% for $\beta = 0.75$. In general, the kernel densities of both villages are centered and the performance of the traditional mean WTP estimator is relatively good. This is not the case with data from Village 3. The mode of the estimated mean WTP distributions for the models with $\beta = 0.5$ and $\beta = 0.75$ are located significantly to the left of the true mean WTP. The coefficient of variation for $\beta = 0.5$ is 14% and for $\beta = 0.75$ is 52%. This result demonstrate how unpredictable the theoretical bias of the standard estimator is when WTP is influenced by real world social networks. Drawing conclusions from the network measures

[16] Refer to Banerjee et al. (2011) for a detalied description of the data.

[17] A *node* corresponds to a respondent. The *degree* of a node is the number of connections of the node. The *average path length* is the average distance between any two nodes in the network. The *betweenness* centrality captures how important a node is in terms of connecting other nodes. The *closeness* centrality tracks how easily a node can reach other nodes. The *density* is the average degree divided by $n - 1$. The *transitivity* measures the probability that the adjacent nodes of a node are connected. The *diameter* of a network is the largest distance between any two nodes. Refer to Jackson (2008) for a detailed explanation of these network measures.

of Table 2, one would think that the bias increases with the size of the real world network, and decreases with their transitivity. We are, however, unable to draw firm conclusions from only three networks. Future work is needed to explore this issue. For instance, with more network data, identification of the sensitivity of the standard approach to the network can be accomplished with a regression of the coefficient of variation on characteristics of the networks.

5 Conclusion

There is empirical evidence suggesting that social utility may be an important component of non-market values.[18] Social networks are natural channels for social preferences to operate through. However, current stated preference approaches to estimation of non-market values do not explicitly accommodate possible social network effects.

This paper builds a network model of random willingness to pay to discuss the consequences of ignoring social network effects in standard approaches for estimation of non-market values using dichotomous choice data. In our framework, the probability of yes/no response is governed by the reduced form equation of the WTP model with networks. The econometric challenge is that, with network effects, the error term of the reduced form model is not homoscedastic. In fact, the variance-covariance matrix of the reduced form model is not diagonal. The reduced form errors are not independent even when the error term of the structural model is homoscedastic and not correlated. This complex variance-covariance matrix structure is induced by the social network that correlates the WTP of a respondent to that of her friends.

We use Monte Carlo experiments to investigate the performance of standard approaches when the data generating process involves a network. We find that the density of Erdos-Renyi networks does not influence the bias of traditional estimates. Also, when respondents' importance is correlated with private WTP, estimates from the traditional approach that

[18]For instance, McConnell (1977) finds that lot of teenagers at a beach make it more attractive to other teenagers. Timmins and Murdock (2007) show that ignoring congestion leads to an understatement of more than 50% of the value of a recreation fishing site. Morey and Kritzberg (2010) use a choice experiment to demonstrate that the presence of a companion can significantly change the value of mountain bike trails.

ignores the network structure have coefficients of variation that can reach 308% of the true WTP value. In addition, standard approaches are not reliable when the simulations use data collected by Banerjee et al. (2011) on social networks of three villages in rural India. Finally, in all simulations, the bias monotonically increases with the strength of the network effect.

With social network effects, the characteristics of the reduced form error term invalidate maximum likelihood estimation based on standard probit or logit models. This presents a great challenge for welfare analysis. Stated-preference valuation approaches are valuable because they can provide rich information about the distribution of WTP. Clearly, a better understanding of the WTP variation allows policymakers to design better public projects. However, failure to account for network effects in the widely used probit models make it impossible to consistently estimate marginal effects. Future work should focus on the development of estimation approaches to overcome these difficulties, allowing researchers to rely on data from the mostly used SP elicitation format even when responses are influenced by social network effects.

Appendix

Figure 3: Kernel Estimates for MC trials with Low Density Erdos-Renyi Networks

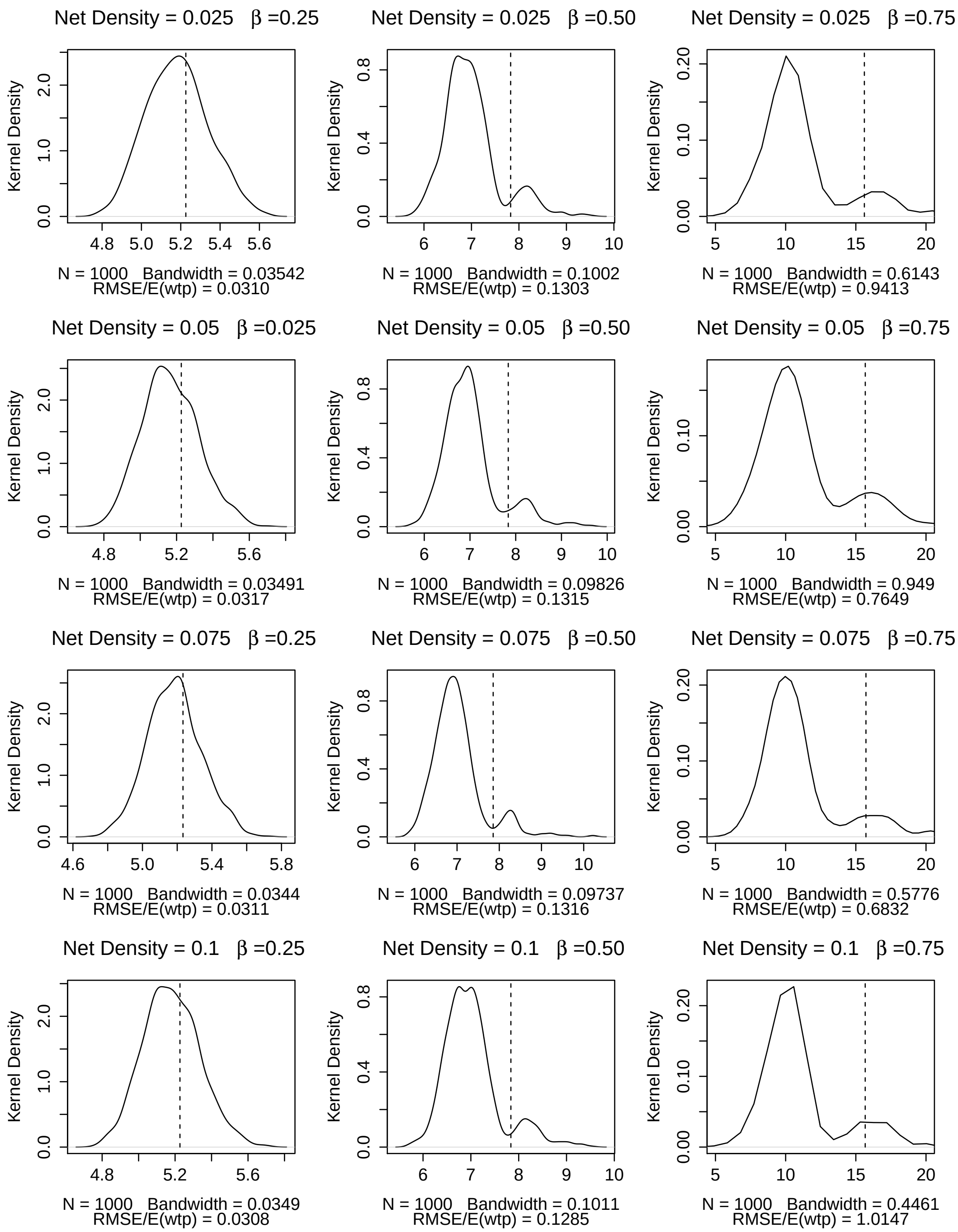


Figure 4: Kernel Estimates for MC trials with High Density Erdos-Renyi Networks

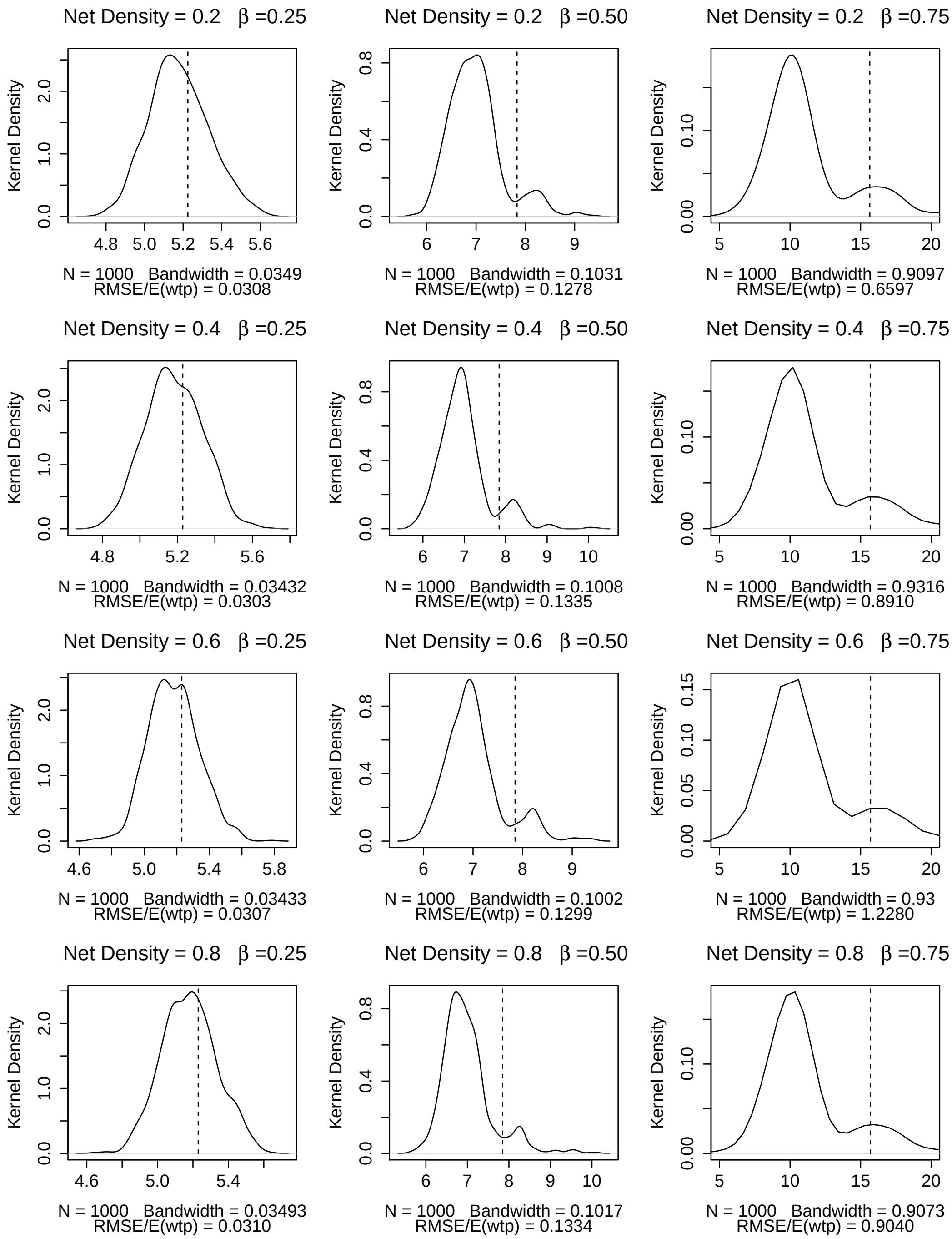


Figure 5: Kernel Estimates for MC trials with Network Importance Matching **X**

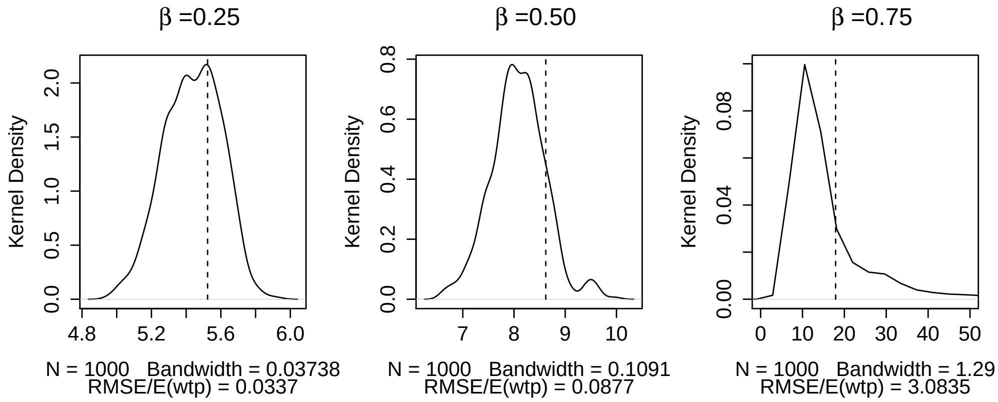


Figure 6: Kernel Estimates for MC trials - Banerjee et al. - Village 1

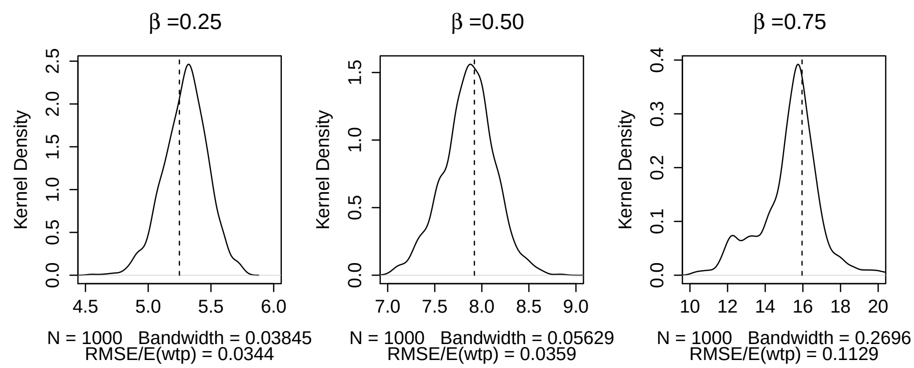


Figure 7: Kernel Estimates for MC trials - Banerjee et al. - Village 2

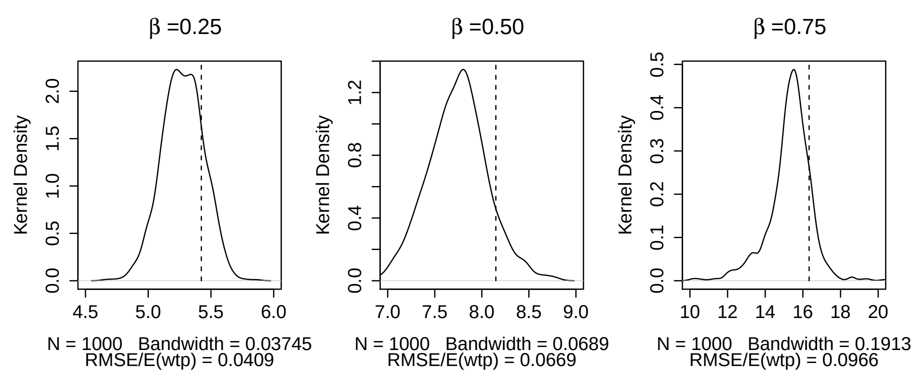
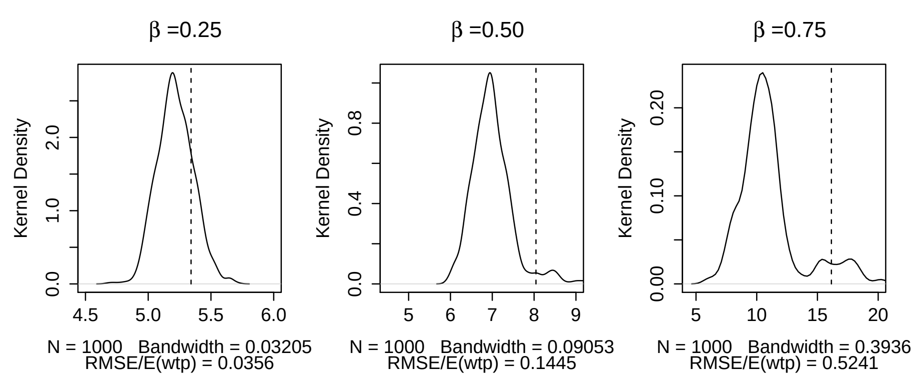


Figure 8: Kernel Estimates for MC trials - Banerjee et al. - Village 3



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