

# 1        **Nonparametric Estimation of a Primary Care Production Function in Urban Brazil\***

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27 **Abstract**

28 **Background** The Brazilian public health system is one of the largest health systems in the world, with a  
29 mandate to deliver medical care to more than 200 million Brazilians. The objective of this study is to  
30 estimate a production function for primary care in urban Brazil. Our goal is to use flexible estimates to  
31 identify heterogeneous returns and complementarities between medical capital and labor.

32 **Methods** We use a large dataset from 2012-2016 (with more than 400 million consultations, 270  
33 thousand physicians, and 11 thousand clinics) to nonparametrically estimate a primary care production  
34 function and calculate the elasticity of doctors' visits (output) to two inputs: capital stock (number of  
35 clinics) and labor (number of physicians). We benchmark our nonparametric estimates against estimates  
36 of a Cobb-Douglas (CD) production function. The CD model was chosen as a baseline because it is  
37 arguably the most popular parametric production function model. By comparing our nonparametric  
38 results with those from the CD model, our paper shed some light on the limitations of the parametric  
39 approach, and on the novelty of nonparametric insights.

40 **Results** The nonparametric results show significantly heterogeneity of returns to both capital and labor,  
41 depending on the scale of operation. We find that diseconomies of scale, diminishing returns to scale,  
42 and increasing returns to scale are possible, depending on the input range.

43 **Conclusions** The nonparametric model identifies complementarities between capital and labor, which is  
44 essential in designing efficient policy interventions. For example, we find that the response of primary  
45 care consultations to labor is steeper when capital level is high. This means that, if the goal is to allocate  
46 labor to maximize increases in consultations, adding physicians in cities with a high number of clinics is  
47 preferred to allocating physicians to low medical infrastructure municipalities. The results highlight how  
48 the CD model hides useful policy information by not accounting for the heterogeneity in the data.

49 **Keywords** Primary Care, Public Healthcare Investment, Returns to Capital and Labor, Heterogeneity,  
50 Nonlinearities, Complementarities.

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## 56 **1 Background**

57 Primary health care represents a broad approach to the promotion of individual and societal health and  
58 well-being, and as such it includes health services employed to deliver prevention, treatment,  
59 rehabilitation, and palliative care. According to the World Health Organization (WHO), the delivery of  
60 quality primary care can have significant short-term impacts in reducing risk factors and poor health  
61 conditions [1]. The WHO recognizes that health systems based on primary health care are of paramount  
62 importance in achieving sustainable health goals. This architecture is especially important in developing  
63 countries, where primary care systems often need to be further developed. WHO works with many  
64 countries to implement primary health care policies that integrate health-promoting and preventive  
65 interventions thereby reducing health care delivery costs and improving efficiency through lower  
66 hospital admissions.

67 Many developing and middle-income countries have been struggling to maintain adequate  
68 levels of public services in light of increasingly tighter budget constraints. For example, since 2010, Latin  
69 American countries have experienced declining GDP per capita growth rates, which dropped from 4.67%  
70 to -0.44% in 2018. In fact, the region has been averaging a negative growth rate since 2014 [2]. This  
71 trend highlights the struggle of financing improvements in standards of living when wealth grows at a  
72 rate slower than that of population. A study by Varela and co-authors shows that only 6% of the  
73 municipalities in Sao Paulo, the largest Brazilian state, efficiently allocate health care expenditures to the  
74 delivery of primary care [3]. Lobo et al. examine a sample of 104 Brazilian teaching hospitals finding  
75 similar results; only 5% of the hospitals efficiently allocate resources [4].

76 As population grows, so does the demand for health care. In this context, it is essential for  
77 managers and policy makers to carefully understand the determinants of medical services uptake. One  
78 approach is to estimate health care production as a function of capital infrastructure in the healthcare  
79 system and the availability of health professionals. Such estimates allow for an evaluation of the returns  
80 to investments in capital and labor. Moreover, it is possible, and even probable, that the outcomes of  
81 health care investments are nonlinear, and the impact of health policies may vary depending on the  
82 scale of capital and labor. For example, it is possible for investments on medical personnel to have  
83 differentiated impacts on output as we move along the distribution of both labor and capital. Are  
84 returns to labor higher when labor is low? Are returns to labor investments a function of the stock of  
85 capital? Having an approach to answer these types of questions is imperative for the efficient design of  
86 health policy.

87           The goal of this paper is to estimate a primary care production function for urban Brazil. We  
88 employ nonparametric methods to estimate the elasticities of the uptake of primary health care services  
89 to capital infrastructure (number of clinics) and labor (number of physicians). We contrast our approach  
90 with a popular parametric baseline model to highlight the advantages of nonparametric approaches,  
91 and to show how the more flexible nonparametric estimates can provide additional support to  
92 evidence-informed public policies.

### 93 *Literature and Contributions*

94           Examination of health production using parametric and nonparametric approaches has been an  
95 ongoing and fruitful area of research in the health economics and policy fields. The literature is large and  
96 evolved in many directions. For example, the efficiency of health care delivery systems has been studied  
97 using parametric models such as the Stochastic Frontier Approach [5]–[7] and nonparametric models  
98 such as Data Envelopment Analysis [8]–[10]. Some papers explore the benefits of parametric estimation  
99 trying to limit the disadvantages of the parametric assumption by using flexible functional forms to  
100 estimate primary care or health production functions [11], [12]. Other papers examine how the choice  
101 of functional form affect parametric estimation of health care technical efficiency [13]. Health  
102 economics research has also compared parametric and nonparametric methods [14],[15].

103           We add to this literature by using both parametric and nonparametric approaches to estimate  
104 elasticities of primary health care delivery in a large urban setting: the Brazilian public health system.  
105 Our contribution is twofold. First, by studying primary care in Brazil we are examining one of the largest  
106 public health systems in the world, with a mandate to deliver universal health care free of charges to  
107 more than 200 million individuals. We examine the five-year period from 2012 to 2016 where Brazil  
108 faced unfavorable economic conditions. As the country (and many others in the region) continues to  
109 struggle to revert the economic environment, data-driven and evidence-based policy recommendations  
110 are in high demand.

111           Second, we contribute by contrasting simple parametric estimates with a set of rich estimates  
112 from nonparametric local-linear models. Our application takes advantage of large datasets (with millions  
113 of observations) to mitigate issues related to the curse of the dimensionality (although at the expense  
114 of computing time). Our paper highlights the insights that nonparametric estimates are able to deliver  
115 regarding heterogeneity, nonlinearities, and cross-effects. In doing so, the paper showcases the  
116 potential of local-linear models in informing public health policy.

117

118 *The Brazilian Health System*

119 The Brazilian Unified Health System (Sistema Único de Saúde – SUS) is a public health system created in  
120 1989 that offers access to health care, free of charge, to all Brazilians. SUS serves a population of more  
121 than 200 million people, ranking among the largest government funded and managed health care  
122 systems of the world. SUS network is responsible for all levels of health care, including all levels of  
123 medical care, laboratory and diagnostic care, physical and occupational therapy, nutritional support,  
124 pharmaceutical care, etc.

125 While all Brazilians are covered by SUS, private medical services and health insurances are also  
126 available for purchase in Brazil. Table 1 shows total Brazilian population and the number of individuals  
127 with supplemental private medical insurance. The data show that Brazilians increasingly depend of  
128 publicly provided health care. In the 2012-2016 period, population growth paired with a decrease in  
129 private insurance enrolment led to the coverage of the Brazilian population with supplemental private  
130 health insurance to decrease from 15.1% to 12.8%.

131 **Table 1: Brazilian Population with Private Medical Insurance**

Year	Total Population	Private Medical Insurance	Private Coverage
2012	193,946,886	29,307,255	15.1%
2013	201,032,714	29,929,896	14.9%
2014	202,768,562	30,449,912	15.0%
2015	204,450,049	28,408,062	13.9%
2016	206,081,432	26,421,183	12.8%

Source: ANS ([www.ans.gov.br/perfil-do-setor/dados-gerais](http://www.ans.gov.br/perfil-do-setor/dados-gerais)) and IBGE ([www.ibge.gov.br](http://www.ibge.gov.br)). Data excludes dental coverage.

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133 **2 Methods**

134 Both parametric and nonparametric approaches can be used to examine health care delivery  
135 productivity and technical efficiency. In productivity studies the interest lies mainly on returns to input  
136 usage, or elasticities. Such estimates are valuable from a policy perspective as they can guide the  
137 allocation of health care resources. In efficiency studies, researchers are interested in measuring how  
138 producers deviate from an estimate of the production function, viewed as the state-of-the-art  
139 technology frontier. Therefore, efficiency studies are useful in informing policies with a focus on  
140 minimizing waste [16].

141           Regardless of the focus, i.e. production or efficiency, these studies require the estimation of a  
142 production function, which begs the question of which method to use. An important methodological  
143 decision is choosing between parametric and nonparametric approaches. There is no consensus in the  
144 literature and pros and cons have been reported about both methods. Approaches based on parametric  
145 functions are simple and can be easily implemented. Under the proper assumptions, parametric  
146 approaches have desirable statistical properties (e.g. fast convergence rates, which is important in small  
147 samples). Nonparametric approaches are more flexible as no functional form is pre-specified and the  
148 shape of the relationship between health output and inputs is determined by the data. On the  
149 downside, nonparametric estimators have low convergence rates and require larger amounts of data to  
150 deliver estimation errors equivalent to those from correctly specified parametric counterparts [16].

### 151 *Model*

152 We conceptualize the delivery of medical services in terms of a medical care production function [17].  
153 We consider the following random production function for healthcare delivery:

$$154 \qquad Y = f(K, L) + \varepsilon \qquad (1)$$

155 where  $Y$  is the delivery of medical services (output) and is determined by two components. The first  
156 component is deterministic and depends on health care inputs, while the second is a random  
157 component. The deterministic portion of output is given by a production function  $f$  that depends on  
158 capital  $K$  and labor  $L$ . The random term  $\varepsilon$  captures unobserved determinants of output and is assumed  
159 to be zero-mean.

160           Our goal is to use data on  $Y$ ,  $K$ , and  $L$  to estimate the elasticities of primary care delivery with  
161 respect to capital and labor, which are determined by, respectively,

$$162 \qquad \frac{\partial Y}{\partial K} \frac{K}{Y} \quad \text{and} \quad \frac{\partial Y}{\partial L} \frac{L}{Y}.$$

163 We are also interested in examining estimates of the conditional mean of output,  $E(Y|K, L)$ . These  
164 estimates allow us to visualize partial prediction plots, which can be of great value to policy makers.  
165 These types of plots are a simple way to illustrate nonlinearities and, as a result, they help inform policy  
166 by highlighting complementarities between capital and labor (see Figure 2).

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168

169 *Estimation*

170 Our approach follows closely the production function estimation described by Henderson and Parmeter  
171 [18].<sup>1</sup> We consider two approaches to estimate a cross-city primary care production function in Brazil.  
172 The first is a parametric approach based on a Cobb-Douglas production function. The second is a  
173 nonparametric approach where no functional form is specified for primary care production.

174 *Parametric model*

175 This section presents a typically used parametric production model to establish a baseline for  
176 comparison with the nonparametric estimates. The Cobb-Douglas production function is arguably the  
177 most common parameterization of production in the literature and has been used to model production  
178 processes for more than one hundred years [19]. For the case of two inputs, capital and labor, the Cobb-  
179 Douglas model with an additively separable error term assumes the primary care function (1) takes the  
180 form

181 
$$Y = AK^\alpha L^\beta + \varepsilon$$

182 where  $A$  is a technology parameter,  $\alpha$  is the elasticity of primary care output with respect to medical  
183 capital, and  $\beta$  is the elasticity of output with respect to the number of physicians. The typical approach  
184 to estimate our parameters of interest  $\alpha$  and  $\beta$  is to log-linearize the model and use Ordinary Least  
185 Squares (OLS) to estimate the parameters in a log-linear regression. Nevertheless, it has been shown in  
186 the literature that such an approach can introduce bias [18], [20]. To avoid such a bias, we estimate  $\alpha$   
187 and  $\beta$  using Nonlinear Least Squares (NLS).

188 The NLS estimation procedure is as follows. To simplify notation, let  $f(X, \theta) = AK^\alpha L^\beta$ , where  
189  $X = (K, L)$  and  $\theta = (A, \alpha, \beta)$ . The nonlinear least squares estimator  $\hat{\theta}$  is the value of  $\theta$  that minimizes  
190 the sum of the squared residuals [21]:

191 
$$\min_{\theta} (Y - f(X, \theta))'(Y - f(X, \theta)).$$

192 The problem can be solved numerically using the Gauss–Newton algorithm, and standard errors are  
193 computed using a wild bootstrap procedure [14].

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<sup>1</sup> Henderson and Parmeter [18], in Chapter 5, offer further details and a comprehensive discussion of alternative parametric (Constant Elasticity of Substitution, Generalized Quadratic, and Generalized Leontief) and nonparametric (Local-Constant and Local-Polynomial) models.

195 *Nonparametric model*

196 Consistent parametric estimation of the elasticities of capital and labor relies on the assumption that the  
 197 parametric functional form chosen, in many cases the Cobb-Douglas, is the correct or true functional  
 198 form. However, there is no consensus in the health economics literature regarding the correct functional  
 199 form for the delivery of primary health care. Nonparametric estimation of function (1) allows us to avoid  
 200 the bias of incorrectly imposing a certain parametric shape to the relationship between inputs  $K$  and  $L$   
 201 and output  $Y$ . In fact, one of the main advantages of the nonparametric approach is that it recovers the  
 202 relationship between inputs and output directly from the data.

203 We use a Local-Linear Least Squares model (LLLS) to approximate the function  $f(K, L)$  in  
 204 equation (1)<sup>2</sup>. The LLLS estimator is perhaps the most popular nonparametric regression estimator. To  
 205 simplify notation, let  $X$  denote the input matrix  $(K, L)$ . The LLLS estimator fits a line on the  
 206 neighborhood of a point  $X_0$ , where the concept of “neighborhood” is determined by a bandwidth vector  
 207  $h$ . The estimator re-writes the original model by considering a Taylor approximation around  $X_0$ :

208 
$$Y = f(X_0) + (X - X_0)\beta(X_0) + \varepsilon,$$

209 where  $\beta$  is the gradient and is treated like a parameter to be estimated, and so is  $f(X_0)$ . Denoting  $a =$   
 210  $f(X_0)$  and  $b = \beta$ , the LLLS chooses  $\theta = (a, b)$  to minimize the weighted sum of squared residuals:

211 
$$\min_{\theta} (Y - \tilde{X}\theta)' K(X, X_0, h) (Y - \tilde{X}\theta),$$

212 where  $\tilde{X} = (1, X - X_0)$ ,  $1$  is a column vector of ones and  $K(X, X_0, h) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{X-X_0}{h})^2}$  is the Gaussian  
 213 Kernel. When the Kernel is the identity matrix, the estimator reduces to the OLS estimator. By using  
 214 Kernel weights we mitigate the effect of poor approximations of points far from  $X_0$ . Standard errors and  
 215 confidence intervals can be computed using a wild bootstrap. To avoid issues with numerical  
 216 optimization and facilitate estimation, we standardize the data by dividing each variable by their mean.

217 The final step required to implement the above LLLS nonparametric estimator is to choose  
 218 bandwidths in  $h$ . Typically, the literature relies on data-driven methods to determine the appropriate set  
 219  $h$  [18]. We use least squares cross validation (LSCV) to determine the bandwidths in  $h$ . The idea is simple  
 220 and relies on choosing  $h$  that minimizes the sum of the squared errors of the model prediction, which in  
 221 turn depends on  $h$ . Formally, the bandwidths in  $h$  minimize  $\sum(Y - \hat{Y}(h))^2$ .

222 In summary, the concept of the LLLS is to fit a linear model around the neighborhood of inputs  
 223  $K$  and  $L$ , where this neighborhood is determined by a bandwidth chosen using cross validation (LSCV).

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<sup>2</sup> Refer to Henderson & Parmeter for a detailed discussion of the LLLS estimator [18].



224 The nonparametric model moves along the distributions of  $K$  and  $L$  estimating local linear regressions,  
225 connecting the predicted outputs (or conditional mean) from these various regressions to generate the  
226 relationship between inputs and output.

227 The nonparametric estimates of the elasticities of capital and labor are computed as

228 
$$\hat{\beta}(K) K/\hat{Y} \text{ and } \hat{\beta}(L) L/\hat{Y},$$

229 respectively, where  $\hat{\beta}(K)$  is the gradient of the conditional mean with respect to capital,  $\hat{\beta}(L)$  is the  
230 gradient of the conditional mean with respect to labor, and  $\hat{Y}$  is the fitted value. Standard errors for  
231 elasticities, returns to scale, the predictions are computed via wild bootstrap as it is consistent under  
232 both homoskedasticity and heteroskedasticity [18]. Estimation was done using the R software [22] using  
233 codes provided by Henderson and Parmeter [18].

#### 234 *Data*

235 Our analysis is based on a sample of SUS users from the 100 largest cities in Brazil. The data contains  
236 information about SUS medical care delivery, physical medical infrastructure, and supply of healthcare  
237 professionals, at the city-level, monthly, from 2012 to 2016. Preliminary examination of the data  
238 revealed that the two largest cities (São Paulo and Rio de Janeiro) represent extreme values when  
239 compared against the remainder 98. We therefore exclude these two cities from our analysis. Our  
240 working sample offers a fair representation of urban Brazil. Collectively, the 98 cities considered in this  
241 study account for approximately one third of Brazil's total population.

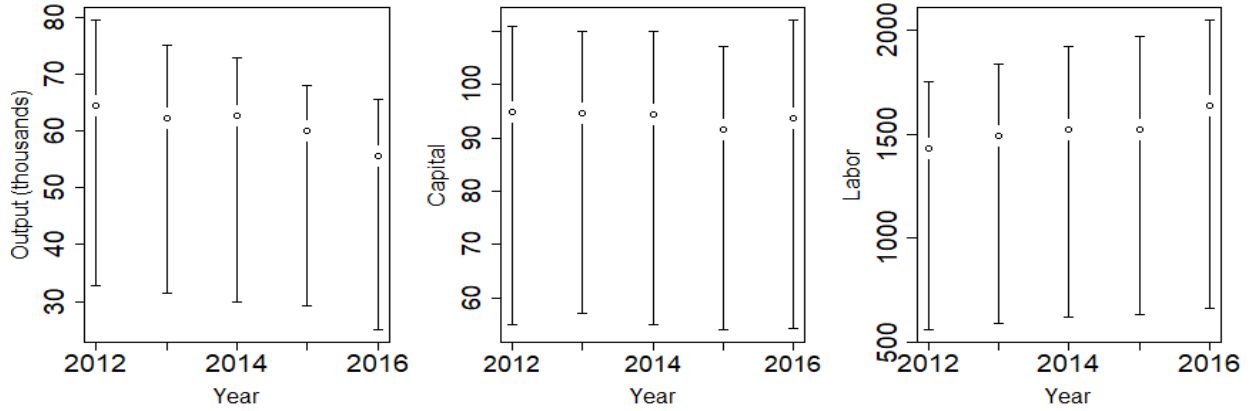
242 The data comes from DATASUS – SUS IT department. DATASUS has several SUS databases that  
243 are publicly available for download.<sup>3</sup> Our output data were collected from the SIA database -- System of  
244 Outpatient Information (Sistema de Informação Ambulatorial – SIA). We measure primary care output  
245 as the number of doctor's visits in the SUS system, for each city-month observation. The number of  
246 patient visits (per unit of time) is a typical measure of primary care output [11]. Our input data are from  
247 the CNES database -- National Registry of Healthcare Facilities (Cadastro Nacional de Estabelecimentos  
248 de Saúde – CNES). Our proxy for health capital infrastructure is the city's number of clinics and similar  
249 health care delivery units. The city's stock of labor is measured as the number of physicians working in  
250 the city's SUS system.<sup>4</sup>

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<sup>3</sup> SUS data are available online at [datasus.saude.gov.br/informacoes-de-saude-tabnet](http://datasus.saude.gov.br/informacoes-de-saude-tabnet).

<sup>4</sup> Refer to Electronic Supplemental Material for further details on measurements of capital, labor, and output.

251 In the 98 cities considered in this study, from 2012-2016, the sample contains 274,175 distinct  
 252 physicians, 11,203 distinct clinics, and 407,259,570 primary care consultations. Figure 1 shows the (city-  
 253 month) average number of consultations, clinics, and doctors, along with the interquartile range (IQR),  
 254 by year. The data show that the representative city-month observation hovers around 60 thousand  
 255 consultations (with a slight decline from 2012 to 2016), just under 100 clinics (steady trend in the  
 256 period), and approximately 1,500 doctors (with a slight upward trend).



257 **Figure 1: Mean and IQR of output, capital, and labor, by year**

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260 **3 Results**

261 We start by reporting the elasticities of capital and labor estimated by the parametric model. The  
 262 simplicity of the Cobb-Douglas model is a feature that makes such a parametrization attractive and  
 263 contributes to the popularity of this model. On the downside, the Cobb-Douglas model does not allow  
 264 the elasticities to vary along different levels of capital and labor. The model’s functional form leads to a  
 265 single elasticity measure for each input, and therefore does not account for nonlinearities and cross-  
 266 effects.<sup>5</sup>

267 Table 2 shows the Cobb-Douglas elasticities estimates along with their standard errors. We find  
 268 elasticities of output with respect to capital and labor of similar magnitude. The elasticities are precisely  
 269 estimated, as indicated by the narrow standard errors. Both inputs have elasticity around 0.38, which  
 270 suggests diminishing returns to scale in the order of 0.76. As a result, we find that primary care delivery  
 271 increases by less than the proportional increase in both medical capital infrastructure and labor

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<sup>5</sup> It can be easily verified that  $\left(\frac{\partial AK^\alpha L^\beta}{\partial K}\right)\left(\frac{K}{AK^\alpha L^\beta}\right) = \alpha$  and  $\left(\frac{\partial AK^\alpha L^\beta}{\partial L}\right)\left(\frac{L}{AK^\alpha L^\beta}\right) = \beta$ .

272 requirements. The Cobb-Douglas results suggest that if policy makers double capital and labor, primary  
 273 care output will less than double and increase by a factor of 1.5.

274

275

**Table 2: Estimates of the Cobb-Douglas model**

Elasticities	
Capital	0.378 (0.016)
Labor	0.381 (0.015)
Returns to Scale	
	0.759 (0.013)

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Note: bootstrapped standard errors are in parenthesis (based on 400 replications).

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As discussed above, consistency of parametric results relies on strong assumptions. To be unbiased, the Cobb-Douglas functional form must be the appropriate functional form for primary health care production. That would imply constant elasticities, and there exists no evidence that this is the case in primary care delivery.

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Next, we use nonparametric models to obtain estimates that are not plagued by parametric misspecifications. The elasticities estimates are driven by the data and are allowed to vary along the distribution of capital and labor. As a result, the nonparametric approach captures nonlinear effects allowing policy makers to have a deeper understanding of the heterogeneity in returns to scale. It is possible, for example, to capture diminishing returns to scale in some input range and increasing returns to scale in another.

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We estimate a nonparametric LLLS model using LSCV to select bandwidths. Table 3 reports the bandwidth estimates. We find bandwidths of 0.037 and 0.453, for capital and labor, respectively. For both values, the estimates are significantly lower than the upper bound suggested in the literature of two times the standard deviation of the corresponding input [23]. This indicates that both capital and labor have a nonlinear effect on output.

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**Table 3: LSCV Bandwidths**

	Bandwidth	Upper Bound
Capital	0.037	1.881
Labor	0.453	3.716

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296 The nonparametric model produces observation-specific elasticities. Table 4 summarizes results  
 297 by showing the LLS elasticities estimates at their mean, 25<sup>th</sup> quantile, median, and 75<sup>th</sup> quantile. The  
 298 table reveals several important lessons. First, focusing at the mean, while the nonparametric estimate of  
 299 the elasticity of capital is similar to that of the parametric model, the LLS estimate of the elasticity of  
 300 labor is significantly smaller than the Cobb-Douglas counterpart. Table 4 shows that mean returns to  
 301 capital (0.391) are more than seven times larger than mean returns to labor (0.050). That has a  
 302 significant effect on the nonparametric estimates of returns to scale. The nonparametric model  
 303 estimates diminishing returns to scale of 0.441, i.e. returns diminishing stronger than those of the  
 304 parametric model. These estimates suggest that doubling input usage leads to an increase in output of a  
 305 factor of 0.88 (as opposed to 1.5 in the Cobb-Douglas model).

306 **Table 4: Results of the nonparametric model**

	Mean	Quantile of Elasticity Distribution		
		0.25	0.50	0.75
Elasticities				
Capital	0.391 (0.246)	-0.903 (0.260)	0.281 (0.166)	1.174 (0.678)
Labor	0.050 (0.077)	-0.227 (0.062)	0.096 (0.122)	0.380 (0.136)
Returns to Scale	0.441 (0.278)	-0.901 (0.177)	0.532 (0.351)	1.376 (0.246)

307 Note: bootstrapped standard errors are in parenthesis (based on 400 replications).  
 308

309 Next, we find significant heterogeneity on both sets of elasticities. In the 25<sup>th</sup> quantile (i.e. the  
 310 bottom of the distribution), the elasticities of both capital and labor are negative and represent  
 311 diseconomies of scale. This is an indication of the existence of dysfunctional local health systems where  
 312 investments in medical inputs actually drive patients away. Diseconomies of scale in the healthcare  
 313 sector have been reported both in developed countries, e.g. the United States [24], and in developing  
 314 countries, e.g. Turkey [13]. At the median, returns to capital are 28% smaller than the mean, and  
 315 returns to labor are almost doubled.

316 Finally, elasticity heterogeneity becomes even more evident when we examine the top of the  
 317 distribution. In the 75<sup>th</sup> quantile, the elasticity of output with respect to capital is 1.174, which suggests  
 318 increasing returns to capital. Labor continues to exhibit diminishing returns. Comparing parametric and  
 319 nonparametric estimates, only at the 75<sup>th</sup> quantile the LLS estimate of the elasticity of labor is similar to  
 320 that of the Cobb-Douglas model. In general, the 75<sup>th</sup> quantile results indicate that urban Brazil has high  
 321 functioning local health care systems with strong increasing returns to scale.

322           When examining production functions, another interesting way to present nonparametric  
323 findings is to plot the estimate of the conditional mean of output (and its confidence interval) against  
324 one input. When production depends on two inputs, as in our case, this plot needs to be constructed by  
325 holding the other input constant at some (arbitrary) level [18]. The exercise we pursue in this paper is to  
326 plot output estimates obtained using one variable input (capital or labor), holding the other input fixed.  
327 For each case (variable capital or variable labor), we estimate three models that differ by the level of the  
328 fixed input (which is fixed at 25<sup>th</sup>, 50<sup>th</sup>, and 75<sup>th</sup> quantiles). This examination allows us to uncover cross-  
329 effects and assess whether there exist complementarities between the medical capital infrastructure  
330 and the supply of physicians.

331           Results are shown in figure 2. The graphs in left column show the counterfactual exercise of  
332 displaying output prediction as a function of capital, holding labor fixed at the 25<sup>th</sup> quantile (PANEL A),  
333 50<sup>th</sup> quantile (PANEL B), and 75<sup>th</sup> quantile (PANEL C). The right column displays the same exercise for  
334 varying labor. We normalize output and inputs by their mean to avoid issues with numerical  
335 optimizations. As a result, an output (input) value of 1 represents mean output (input). Similarly, a value  
336 of 2 represents a level twice the mean. In addition to facilitate optimization, this normalization simplifies  
337 the scale of the graphs, making them easier to read while retaining economic meaning by allowing for  
338 cross comparisons.

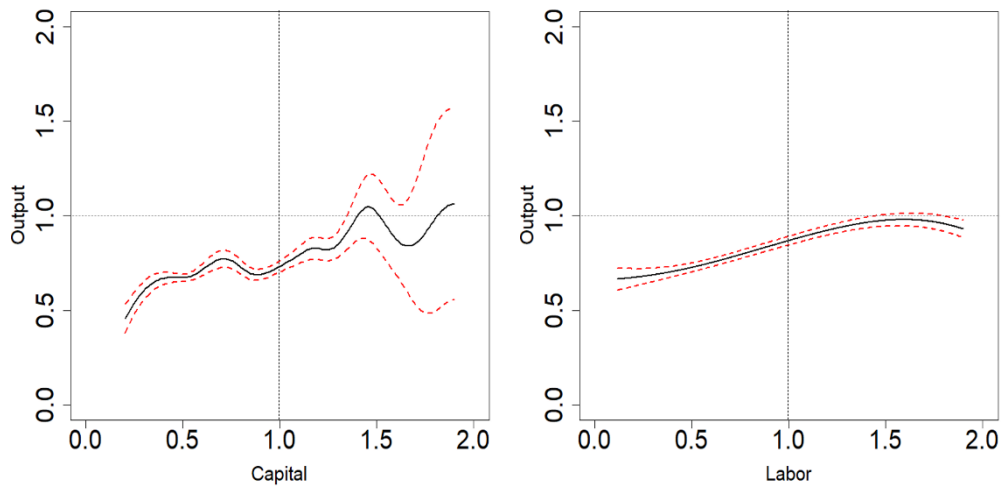
339           The counterfactual plots of output against capital (holding labor constant) are far more volatile  
340 than the ones of output and labor (holding capital constant). This finding suggests that there is more  
341 uncertainty in health care capital investments than in investments to expand the number of physicians.  
342 In general, as input usage increases, output increases. But there are downward-sloping regions,  
343 especially on the left-hand side graphs for high levels of capital. The right-hand side column shows a  
344 similar effect for labor, however with significant less variance and a shallower negative output response  
345 at the upper tail of the labor distribution.

346           The counterfactual analysis complements the previously discussed elasticity analysis. The graphs  
347 allow us to gauge the slope of the production function at different points, which shows information  
348 about the general regions with diseconomies of scale, diminishing returns, and increasing returns. While  
349 results in table 4 show that there are negative elasticities in the bottom of the elasticity distribution  
350 (25<sup>th</sup> quantile), figure 2 suggests that many of these negative results occur at the top of both the capital  
351 and labor distributions.

352

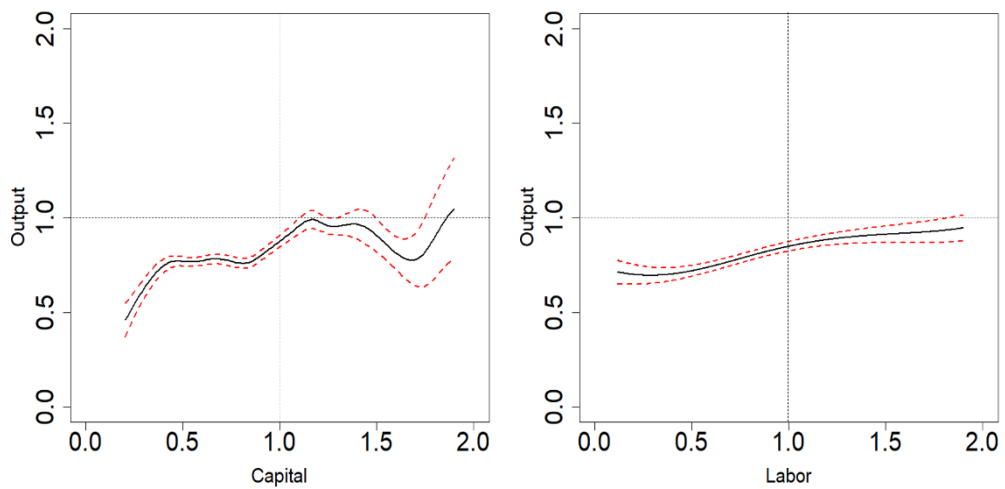
353

PANEL (A) – Prediction holds the other input fixed at the 0.25 quantile



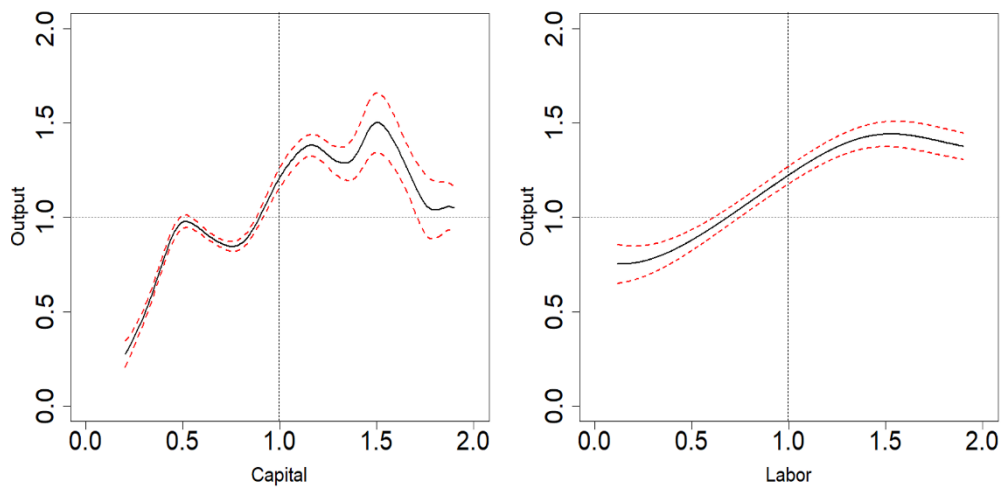
354  
355

PANEL (B) -- Prediction holds the other input fixed at the median



356  
357

PANEL (C) – Prediction holds the other input fixed at the 0.75 quantile



358  
359

Figure 2: Conditional mean output estimates versus a single input

360 In figure 2, a comparison of rows within a column reveals complementarities between capital  
361 and labor, but these are not uniform. For instance, on the right-hand side column, in PANEL A, the entire  
362 prediction of output lies below the mean output (or below 1). The behavior of the conditional mean as  
363 labor varies does not change much when capital increases from the 25<sup>th</sup> quantile to the median (i.e.  
364 comparing right-hand sides of PANELS A and B). One exception is that negative returns to labor at the  
365 top of the labor distribution disappear when capital increases from 25<sup>th</sup> to 50<sup>th</sup> quantile. However,  
366 moving down to PANEL C (i.e. output estimates hold capital fixed at the 75<sup>th</sup> quantile), we see that  
367 output is at a higher level and many points are above the mean. Moreover, output response to labor is  
368 steeper when capital level is high. In general, we conclude that medical infrastructure complements  
369 physicians mainly when capital levels are high, but high capital levels may also induce diseconomies of  
370 scale for expansion of physicians at the top of the labor distribution.

371

#### 372 **4 Discussion**

373 We estimate a primary care production function using data from the public health care systems of the  
374 largest cities in Brazil. In doing so, we compare the insights from the popular Cobb-Douglas parametric  
375 model against those from a nonparametric model. Our results highlight the rich set of policy-related  
376 information that can be generated by flexible nonparametric estimates. With the fast-pace development  
377 of information systems making large datasets available to managers, the application of nonparametric  
378 methods offers flexibility in an environment where the standard “curse of dimensionality” can be  
379 circumvented.

380 Nevertheless, the study has limitations. While the nonparametric model is able to deliver a  
381 variety of interesting insights by exploring the heterogeneity in the data, in our application for primary  
382 care in urban Brazil, we find estimates that are less efficient than those from the parametric model. For  
383 instance, while table 3 reports a parametric elasticity of output to capital of 0.378 with standard error of  
384 0.016, table 4 reports a similar nonparametric mean elasticity of capital of 0.391, however, with much  
385 larger standard error of 0.246.

386 The paper’s output is the city’s number of primary care medical consultations. Although it is not  
387 always necessarily the case that efforts to increase the number of consultations are needed or desired,  
388 it seems reasonable to expect that increases on primary care consultations in urban Brazil are welfare  
389 enhancing. In our sample, the average number of primary care consultations per person per year is 0.4,  
390 which is significantly below the SUS recommendation of 2-3 visits/person/year [25].

391 While the approaches presented in the paper are applied to local health care systems (i.e.  
392 cities), the measurement of health care production and its associated efficiency can also be applied at  
393 the health care delivery unit level. This method would require output and input data at the hospital or  
394 clinic level. Such an approach would offer micro-level information that could inform budgetary  
395 decisions. Here, a word of caution is warranted. The direct application of production and efficiency  
396 estimates to reimbursement frameworks can be problematic as production models are limited by their  
397 ability to capture nuances of the health care delivery process. For example, if output measures are not  
398 adjusted for quality of care, the models may underestimate the return to investments and high-quality  
399 health care facilities could appear to be unproductive [26].

400 Adopting a more aggregated perspective that examines local healthcare systems, our models  
401 estimate elasticities and identify the existence of significant heterogeneities in urban Brazil. Our  
402 approach is able to identify that some local healthcare systems function well, while others do not.  
403 However, our approach does not inform the drivers of these results, nor the reasons and specific  
404 mechanisms underlying undesirable outcomes. In that, this study represents only the first step of a more  
405 comprehensive analysis of primary care. Our methods are able to identify cities with high-return health  
406 systems that may represent investment opportunities, and cities with diseconomies of scale whose local  
407 healthcare sector should be examined more closely. Future research is needed to investigate the role of  
408 characteristics of the city, such as sociodemographic composition (e.g. age, education, income), on  
409 determining the returns to scale in health care.

410

## 411 **5 Conclusions**

412 Many countries struggle with fiscal challenges and must overcome great difficulties to finance health  
413 care systems. This highlights the growing importance of efficient investments in primary health care,  
414 especially in fiscally challenged countries with large health care systems like Brazil. It is, therefore,  
415 fundamental to identify investments opportunities to leverage budgets and achieve maximum  
416 outcomes.

417 Flexible nonparametric production methods can help design health care policy. For the most  
418 part, the nonparametric literature applied to health care production has focused on envelopment  
419 techniques such as Data Envelopment Analysis (DEA) and Free Disposal Hull (FDH) to estimate a health  
420 care production frontier [10], [27]–[32]. However, envelopment estimators are sensitive to outliers and  
421 extreme values and, as a result, may deliver biased estimates of returns to scales, which compromises  
422 the ability to inform health care investments [33], [34].



423 This study shows how nonparametric methods can be used to inform public health policy. We  
424 estimate a LLLS production model that is, by construction, less sensitive to extreme values and outliers  
425 than envelopment estimators. The model is applied to a large dataset of Brazilian cities with more than  
426 400 million consultations, 270 thousand physicians, and 11 thousand clinics, from 2012-2016.

427 We find that, while the results of the typically used CD parametric model suggest that average  
428 returns to medical capital and labor in urban Brazil are similar, the more flexible nonparametric  
429 estimates indicate that average returns to capital are almost 8 times larger than returns to labor. That is,  
430 capital investments promote, on average, higher uptake of primary care services. In other words,  
431 nonparametric results suggest that, on average, expanding the number of clinics, holding the number of  
432 physicians constant, is more effective than expanding the number of physicians, holding the number of  
433 clinics constant.

434 The nonparametric model allows us to go beyond average estimates and explore the heterogeneity  
435 in returns to capital and labor. We find that when the goal is to increase the uptake of primary care  
436 services in Brazil, investments in health care capital are more uncertain than expanding the number of  
437 physicians. Results reveal significant heterogeneity on returns to scale along the distribution of medical  
438 capital and labor, however, returns to capital infrastructure are generally higher than returns to  
439 physicians. Medical infrastructure complements physicians mainly when capital levels are high, but high  
440 capital levels may also induce diseconomies of scale when expanding the number of physicians at the  
441 top of the labor distribution.

442 A recent World Health Organization report shows that global health spending is on an upward  
443 trajectory [35]. This is especially important for Brazil. During the period 2012-2016, Brazil's annual  
444 health spending averages \$1,325 (international dollars) per capita, which ranks Brazil as the 58<sup>th</sup> country  
445 in health spending, globally. This level of investment represents only 28% of that from OECD countries,  
446 or 52% of the investments of Europe and Central Asia [2]. These statistics are striking considering that  
447 Brazil was the 7<sup>th</sup> largest economy in the world in 2012 (behind only U.S., China, Japan, Germany, U.K.,  
448 and France). In fact, despite having economies of similar size, Brazil's health expenditure (per capita) is  
449 only 29% that of France.

450 Currently, the Brazilian federal government faces a tremendous amount of political and popular  
451 pressure to increase health investments and expand the SUS network. In a scenario of health expansion,  
452 understanding nonlinearities and complementarities between medical inputs is important as it allows  
453 policy makers to predict the returns of increasing the availability of an input, depending on the profile of

454 each local market (e.g. scale and level of input complementarity). This information can be harnessed to  
455 determine, for instance, priority regions where investments have higher productivity.

456

## 457 **List of abbreviations**

458 CD: Cobb-Douglas

459 CNES: National Registry of Healthcare Facilities

460 DATASUS: SUS IT department

461 DEA: Data Envelopment Analysis

462 FDH: Free Disposal Hull

463 GDP: Gross Domestic Product

464 IQR: Interquartile Range

465 LLLS: Local-Linear Least Squares

466 LSCV: Least Squares Cross Validation

467 NLS: Nonlinear Least Squares

468 OLS: Ordinary Least Squares

469 SIA: System of Outpatient Information

470 SUS: Brazilian Unified Health System

471 WHO: World Health Organization

472

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